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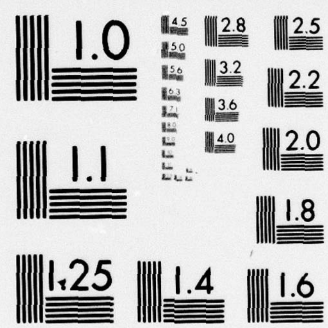

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Research Study 66-6

MANPOWER ROTATION MODELS FOR COMBINED CAREER AND NON-CAREER SYSTEMS

OCTOBER 1966

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6 MANPOWER ROTATION MODELS FOR COMBINED CAREER AND NON-CAREER SYSTEMS

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Research Studies are special reports to military management. They are usually prepared to meet requests for research results bearing on specific management problems. A limited distribution is made--primarily to the operating agencies directly involved.

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FOREWORD

In the MANPOWER OPERATIONS RESEARCH Task, the Statistical Analysis and Research Laboratory develops modeling techniques to aid the solution of personnel management problems relating to the distribution, training assignment, and career progression of personnel in current and future systems. Task effort augments that of the COMPUTERIZED SYSTEMS Task, exploring manpower problems in greater depth, providing further generalizations of models developed to answer specific questions, and extending developed models to other manpower management problems.

The models described in the present Research Study were developed in response to a request from the Office of the Assistant Secretary of Defense, Systems Analysis, for limited assistance in estimating manpower needs. The study represents U. S. APRO's use of mathematical models in the study of manpower problems, and specifically the capability of modeling techniques to respond quickly with static models applicable to specific problems.

The entire research Task is responsive to special requirements of the Deputy Chief of Staff for Personnel and the Office of Personnel Operations, as well as to requirements to contribute to achievement of the objectives of RDT&E Project 2J024701A723, "Human Performance in Military Systems."

J. E. Uhlaner

J. E. UHLANER
Director
USAPRO Laboratories

MANPOWER ROTATION MODELS FOR COMBINED CAREER AND NON-CAREER SYSTEMS

BRIEF

Requirement:

were developed

To develop and present two simplified mathematical models for estimating manpower requirements in the career and non-career Army personnel systems for given conditions of tour duration and allocation quotas.

Procedure:

Two simplified models were developed and the application demonstrated in two examples.

End Product:

Models for the combined career and non-career (one-term) Army personnel systems were prepared. The first may be used as a tool in estimating manpower requirements when long-tour and CONUS categories are pooled in a single rotation base, the second when it is desirable to study the long-tour overseas category separately.

Utilization of End Product:

Depending on the degree of grossness acceptable in manning estimates, one of the two models may be used, either applied to the total system or to MOS subsystems.

MANPOWER ROTATION MODELS FOR COMBINED CAREER AND NON-CAREER SYSTEMS

Manpower rotation policy models have proved useful in evaluating aspects of Army policy concerning tour length, allocation of strength, and manpower requirements. The first U. S. APRO report on the development and application of manpower models, published in July 1965, dealt with personnel flow considerations relative to the reduction of turbulence (1). Subsequent accomplishment includes the development of analytic models incorporating several levels of complexity (2) and a series of nomograms for Army manpower policy evaluation, based on selected models (3). Several computer programs have been written and used, and a simulation study of a dynamic flow model for a changing system was completed (4). Related projects are in progress.

In making a mathematical model for any complex system, compromise between realistic representation and analytic simplicity is usually necessary. Modeling the Army personnel rotation system is no exception. A model should be as simple as possible and still reflect the influence of all factors which might significantly change values being studied. A law of rapidly diminishing returns for increased complexity is the general rule with respect to models used in most simulation studies. Dependable data are hard to come by and the simpler models can usually make use of data summaries more reliable than the unaggregated data. The two models reported on here are data-free models as contrasted to data-bound models that are dependent on good estimates of distribution parameters for model variables, as was the case in studies of an optimal allocation system for enlisted personnel.[1] Data-free models are exceptions to the general rule since they are not concerned with how a real system behaves but how a system would behave if certain policies were followed. Under these conditions, a complicated policy requires a complicated model.

MODEL I

The simple static model combines CONUS and long-tour categories into a common replacement pool. So long as the two tours are of equal duration and the proportion of persons assigned to long tour is small compared to that on the short tour, the gross estimates derived can be an adequate approximation.

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[1] Computerized Manpower Systems subtask c "Distribution models for optimizing performance" (USAPRO Work Program, FY 1967). See USAPRO Technical Research Report, "Amount of assignment information and expected performance of military personnel." (in press).

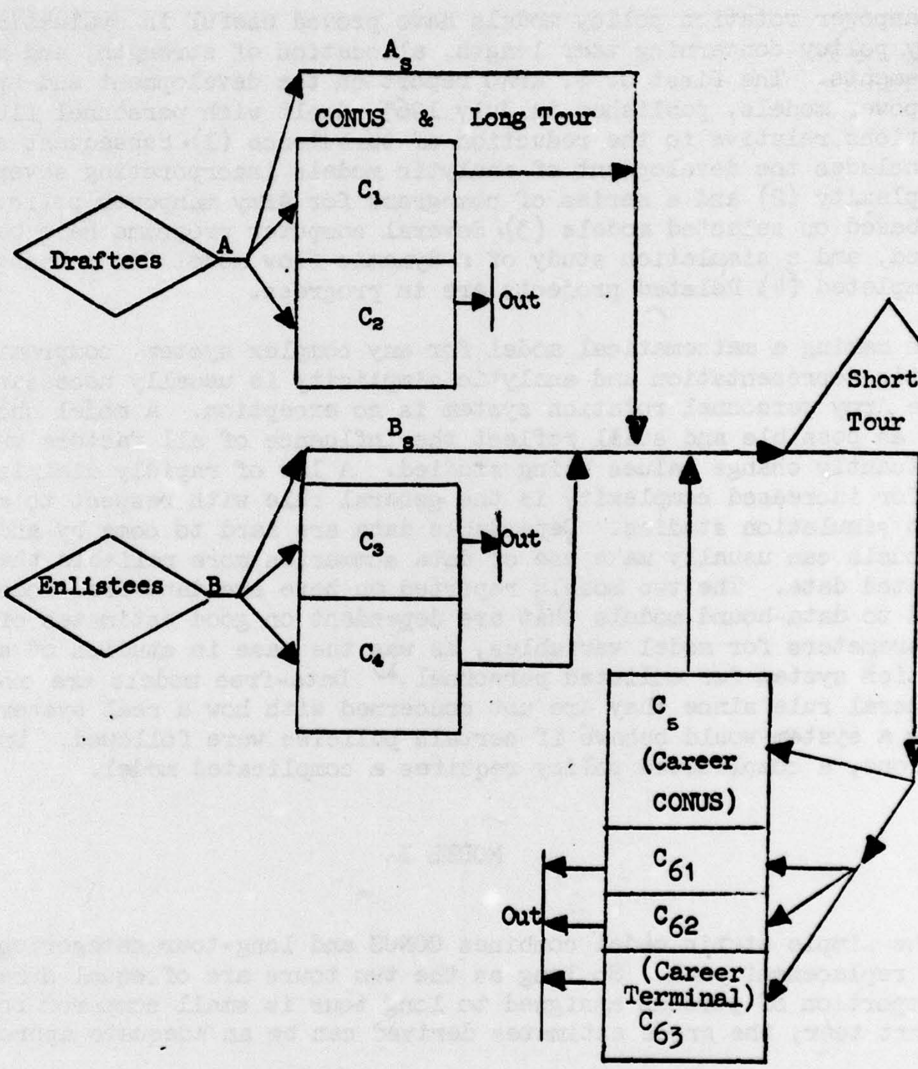

Figure 1. Model of a manpower system which includes one-term persons and multiple-term persons.

The simplified model (Figure 1) was designated especially for use in estimating overall manpower requirements. However, more information can be gained by representing both long and short overseas tours plus the CONUS tour, and by examination of the required proportions of persons on repeated short tours or inter- theater transfer. Further, use of the model separately for each MOS and summing of requirements leads to a more realistic estimate of manning levels than can be obtained by pooling MOS. Model and work examples are presented below:

Notation Used

C_1 = duration of CONUS tour for draftees who serve out their commitment in CONUS but reenlist to join the career Army.

C_2 = duration of CONUS tour for draftees who serve out their term in CONUS then leave the service.

C_3 = duration of CONUS tour for enlistees who serve out their full term in CONUS then leave the system.

C_4 = duration of CONUS tour for enlistees who stay in CONUS for their full first term and who reenlist in the career Army.

C_5 = duration of CONUS tour for career men.

C_{61} = duration of CONUS tour for draftees who leave the system after their required service following short tour.

C_{62} = duration of CONUS tour for enlistees who leave the Army after their required service following short tour.

C_{63} = duration of terminal CONUS tour for career men.

S = duration of short tour.

D_d = duration of minimum term for draftees (after training).

D_e = duration of first enlistment (also after training).

N_t = total number in the system.

N_s = total number of positions to be filled in short tour.

N_{cs} = number of career slots in short tour.

L_c = number lost from the career Army each month (unit time)

A = input of draftees each month.

B = input of enlistees each month.

L_d = number of draftees lost each month.

L_e = number of enlistees lost each month.

N_i = number in CONUS category i

Note: In this model CONUS is understood to mean the combined rotation base, CONUS and long tour.

Model Development

Assume the system has been in operation long enough to reach a steady state so that the flow in and flow out of a station are equal. The following relationships may be written:

Flow through C_{61} is $A_s(\frac{L_d}{A})$

Flow through C_{62} is $B_s(\frac{L_e}{B})$

Flow through C_{63} is L_c

Flow through C_2 is $(A-A_s)\frac{L_d}{A}$

Flow through C_3 is $(B-B_s)\frac{L_e}{B}$

Flow through C_1 is $(A-A_s)(1-\frac{L_d}{A})$

Flow through C_4 is $(B-B_s)(1-\frac{L_e}{B})$

Flow through C_5 is $N_{cs/S} - L_c$

Input, $A + B$, equals loss, $L_d + L_e + L_c$.

The number in a category is equal to the flow in times the duration of the category.

$$N_t = AD_d + BD_e + N_5 + N_{cs} + N_{63}$$

$$N_5 = \left[\frac{N_{cs}}{S} - L_c \right] C_5$$

$$N_{63} = L_c \, C_{63}$$

$$\text{or } N_t = AD_d + BD_e + \left[\frac{N_{cs}}{S} - L_c \right] C_5 + N_{cs} + L_c \, C_{63} \qquad \text{(I-a)}$$

This final equation is the basis of estimates obtained in the work examples given. It has been simplified by substituting 24 for the CONUS tour duration, C_5, and 12 for short tour, S.

$$N_t = AD_d + BD_e + 3N_{cs} + L_c [C_{63} - 24] \qquad \text{(I-b)}$$

Application to Sample Problems

Suppose the short tour requires the assignment of 210,000 men, 90,000 of whom are career level requirements. First term enlistments are divided equally between selective service inductions and voluntary enlistments; 15% of the first termers stay in the Army (the same number of career men must leave the Army, since we are studying the system in steady state); the desired CONUS tour is 24 months and short tour is 12 months; and D_d = 18 months and D_e = 30 months. Use of Equation I-b gives the following

$$N_t = 18(5000) + 30(5000) + 270{,}000 - 12(1500) = 492{,}000.$$

Notice that no provision has been made to take in any first termers in excess of those required by short tour and C_{63} = 12 months was estimated.

Now, assume the duration of service for inductees is shortened to 15 months (after training).

$$N_t = 15(5000) + 30(5000) + 270{,}000 - 18{,}000 = 477{,}000.$$

Suppose 24 months is a more nearly correct estimate of the average terminal CONUS assignment for career men. Then

$$N_t = 495{,}000.$$

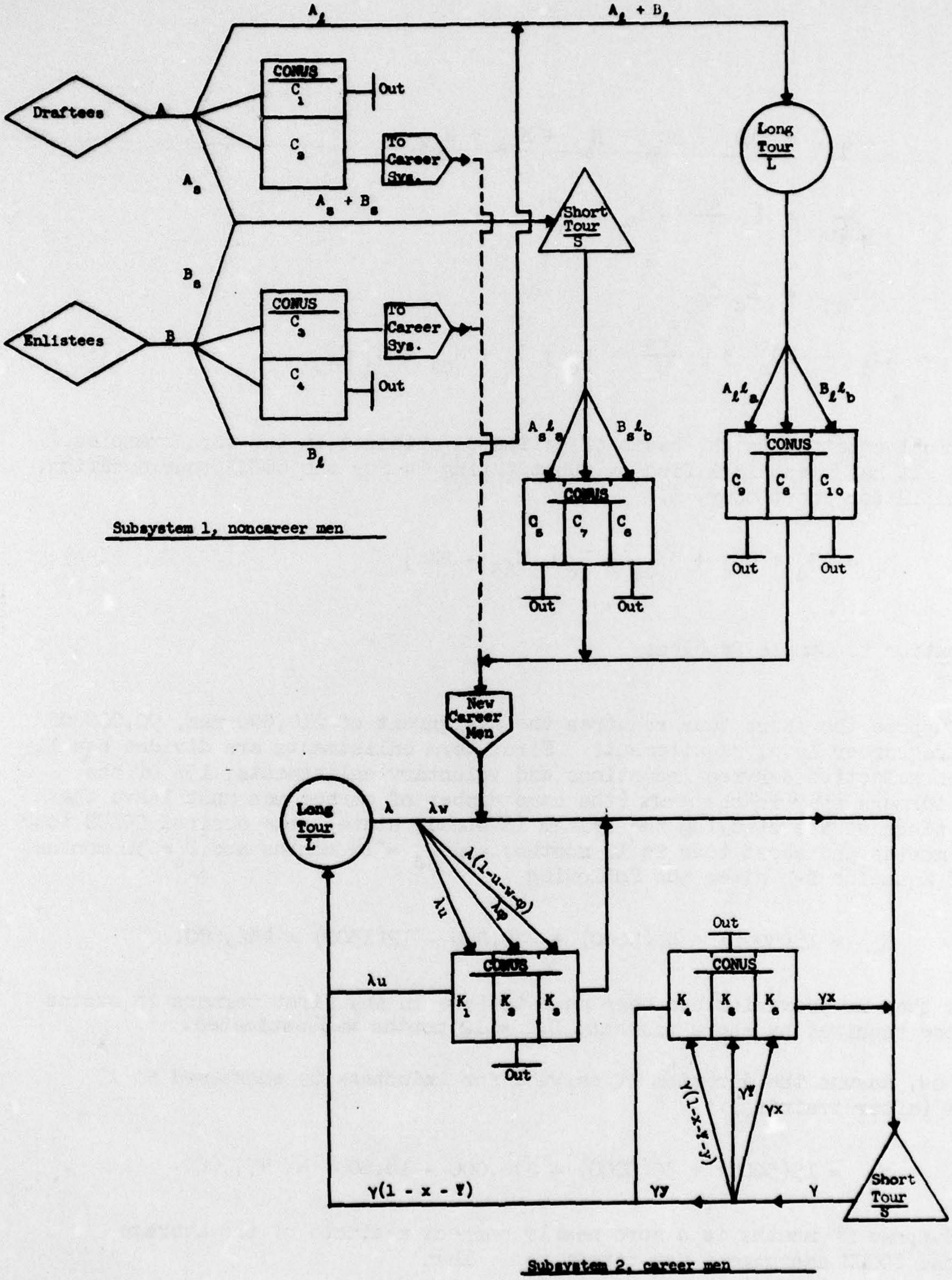

Figure 2. Model of a manpower system with one term and multiple term persons and two categories of overseas assignment.

For a second example, suppose first termers are two-thirds draftees and allow a 10% cushion of input.

$N_s = 210{,}000$ $\quad S = 12$ $\quad C_5 = 24$ $\quad C_{63} = 12$

$N_{cs} = 90{,}000$ $\quad D_d = 18$ $\quad D_e = 18$ $\quad L_c = 1650$

Then, 120,000 is the number of short tour slots to be filled by first-term men.

$[120{,}000 + 12{,}000] \div S = 11{,}000$ per month

$2/3 \times 11{,}000 = 7{,}333$ draftees

$1/3 \times 11{,}000 = 3{,}667$ enlistees

$N_t = 18(7333) + 30(3667) + 3(90{,}000) - 12(1650) = 492{,}000$ (about)

If $D_d = 15$, $N_t = 470{,}000$

If $D_d = 15$ and $C_{63} = 24$, $N_t = 490{,}000$.

MODEL II

The manpower subsystems of one-term and multiple-term personnel can be represented by a model which provides for the study of separate long and short tour requirements and the effect of inter-theater transfers on tour durations. Again, this model is basically a combination of two separate systems, with both distinctive and common problems; however, study of the two as a unit can give meaningful information.

The need to represent a greater number of categories and flows has led to a slightly different notation for the diagrams of this model and for the mathematical relationships following. The assumption is made that losses follow return to the Continental United States and that input to the career system is to short tour. Casualties in the short tour would serve to shorten the average short tour duration and require dummy increases in the flow into the terminal CONUS categories (people staying no months) but no great discrepancies in computations would result. (see Figure 2.)

Required information: Number of 1st term allocations in long and short tours; retention rates for draftees and enlistees; time of transition to career status; length of tours to consider; number of career requirements in

long and short tours; duration of average terminal assignment; duration of tours; policy on inter-theater transfers.

This model may be adapted to solve for possible CONUS tour duration when allocation proportions are used.

Notation Used

A = monthly input of inductees

B = monthly input of enlistees

A_s, A_ℓ = monthly input of inductees to short and long tour, respectively

B_s, B_ℓ = monthly input of enlistees to short and long tour, respectively

C_i for i = 1 to 10, CONUS tour durations for various flow patterns

(see diagram, Subsystem 1)

L'_d and L'_e, duration of long tour for draftees and enlistees

r_d and r_e, retention rates for draftees and enlistees

K_i for i = 1 to 6, CONUS tour durations for career persons

γ, flow from short tour

λ, flow from long tour

x and u, proportion of flow on back-to-back short tour or long tour

ψ and φ, proportion of flow going out of system

w and y, proportions of flow on inter-theater transfer

N_t = total number in system

N_1 = total number in one-term sub-system

N_2 = total number in career sub-system

${}_cN_c$ = number in CONUS for career sub-system

J = the duration of the noncareer status

Model Development

$$N_1 = AD_D + BD_E + Ar_d(J-D_D) + Br_e(J-D_E) \tag{1}$$

$$N_2 = {}_cN_s + {}_cN_\ell + {}_cN_c \tag{2}$$

$$ {}_cN_c = \lambda u K_1 + \lambda\varphi K_2 + \lambda(1 - u - \varphi - u)K_3 + \gamma(1 - x - \psi - y)K_4 + \gamma\psi K_5 + \gamma x K_6 \tag{3}$$

If $K_1 = K_3 = K_4 = K_6 = K$ and $K_2 = K_5 = T$,

$$ {}_cN_c = [\lambda(1 - \varphi - w) + \gamma(1 - \psi - y)]\, K + (\lambda\varphi + \gamma\psi)T \tag{4}$$

5 Since $Ar_d + Br_e = \lambda\varphi + \gamma\psi$

$$ {}_cN_c = [\lambda(1 - w) + \gamma(1 - y)]\, K + (Ar_d + Br_e)(T - K) \tag{6}$$

$$\gamma = \gamma x + \lambda(1 - u - \varphi) + Ar_d + Br_e \tag{7}$$

$$x = \frac{\gamma - \lambda(1 - u - \varphi) - Ar_d - Br_e}{\gamma} \tag{8}$$

and

$$\text{Min } x = 1 - \frac{\lambda(1 - \varphi) + Ar_d + Br_e}{\gamma} \tag{9}$$

but $Ar_d + Br_e = \gamma\psi + \lambda\varphi$

10 so $\text{Min } x = 1. - \dfrac{\lambda + \gamma\psi}{\gamma} = 1. - \psi - \dfrac{\lambda}{\gamma}$

If $(1 - \psi - \frac{\lambda}{\gamma}) < 0$, $x = 0$ and $u \geq 0$,

If $x = 0$, $u = 1 - \dfrac{\gamma(1 - \psi)}{\lambda}$

Note: γ and λ are flows per unit time expressed in number of men; other lower case Greek and Roman characters are coefficients of flow.

Application to Sample Problem

Suppose 120,000 first term requirements must be met in the short tour, 90,000 career level requirements; 30,000 first term in the long tour, 25,000 career level in the long tour. Tour durations are

$$D_d = 18,\ D_e = 30,\ J = 30,\ K = 24,\ T = 12.$$

Retention rates are both equal to 15%, and there are two times as many draftees each month as enlistees. Find the required size of the total system. Assume there is no intertheater transfer.

Given:

$$\left.\begin{array}{lcl} {}_{A+B}N_s & = & 120{,}000 \\ {}_{c}N_s & = & 90{,}000 \end{array}\right\} \quad {}_{tot}N_s = 210{,}000$$

$$\left.\begin{array}{lcl} {}_{A+B}N_\ell & = & 30{,}000 \\ {}_{c}N_\ell & = & 25{,}000 \end{array}\right\} \quad {}_{tot}N_\ell = 55{,}000$$

$S = 12$

$L = 24$

$J = 30$

$D_d = 18$

$D_e = 30$

$K = 24$

$T = 12$

$r_e = r_d = 15\%$

$w = 0,\ y = 0$

Required: N_{total}

Formulas: $N_{total} = N_1 + N_2$

$$N_1 = A\,D_d + B\,D_e + Ar_d\,(J-D_d) + Br_e(J-D_e)$$

$$N_2 = {}_cN_s + {}_cN_e + {}_cN_c$$

$${}_cN_c = [\lambda(1-w) + \gamma(1-y)]\,K + (Ar_d + Br_e)\,(T-K)$$

$$\gamma = \frac{{}_cN_s}{S}$$

$$\lambda = \frac{{}_cN_\ell}{L}$$

$$A+B = \frac{A+B^{N}s}{S} + \frac{A+B^{N}\ell}{L}$$ Assumes $A_s + A_\ell = A$, $B_s + B_\ell = B$.

$$A = 2B$$

Computations:

$$\gamma = \frac{{}_cN_s}{S} = \frac{90000}{12} = 7500$$

$$\lambda = \frac{{}_cN_\ell}{L} = \frac{25000}{24} = 1042$$

$$A+B = \frac{120,000}{12} + \frac{30000}{24} = 11250$$

$$A = 2B$$

$$3B = 11250$$

$$B = 3750$$

$$A = 7500$$

$$N_1 = AD_d + BD_e + Ar_d(J-D_d) + Br_e(J-D_e)$$

$$N_1 = 7500(18)+3750(30)+7500(.15)\,(12)+3750(.15)(0)$$

$$N_1 = 135{,}000 + 112{,}500 + 13{,}500$$

$$= 261{,}000$$

$${}_cN_c = [\lambda(1-w) + \gamma(1-y)]K + (Ar_d + Br_e)(T-K)$$

$${}_cN_c = [1042 + 7500]\ 24 + (1688)(-12)$$

$$= 205{,}000 - 20{,}250$$

$$= 184{,}750 \text{ or } 185{,}000$$

$$N_2 = {}_cN_s + {}_cN_\ell + {}_cN_c$$

$$= 90{,}000 + 25{,}000 + 185{,}000 = 300{,}000$$

$$N_1 + N_2 = 561{,}000.$$

Suppose we are also interested in knowing the extent of the required repeated short tour assignments in this system.

To evaluate the required proportion on back-to-back short tours, it is necessary to solve for the effective loss rates from

$$Ar_d + Br_e = \lambda\varphi + \gamma\psi.$$

Assume $\varphi = \psi$ and substitute for all known values

$$(7500)(.15)+(3750)(.15) = \psi(7500+1042)$$

$$\psi = .20$$

Now use $\text{Min } x = 1 - \dfrac{1042 + (.20)(7500)}{7500}$

$$= 1 - .34 = .66$$

CONCLUSIONS

Two simplified models of the combined first term and career Army manpower systems have been shown. The first shows only the short or hardship tour and the rotation base. The second has provision for a second category of foreign tour. Either may be used in making very gross estimates of manpower requirements. Accuracy of estimates may be improved by application of the models to homogeneous subsamples of the total system and accumulation of the subtotals.

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